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SELF-SIMILAR MOTIONS OF A RADIATION-HEATED GAS BEHIND THE ABSORPTION-INITIATING SHOCK WAVE FRONT

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When acted on by sufficiently powerful light beams, gases which transmit radiation under ordinary conditions can experience breakdown. A survey of experimental data and theoretical information on this phenomenon will be found in [1], one of whose aspects is the shifting of the absorption zone to meet the oncoming light beam.

The present paper concerns the motion of the gas and heating behind the absorption wave propagating solely as a result of one of the mechanisms noted in [1] (the hydrodynamic mechanism). The mechanism consists essentially in the following: a shock wave begins to propagate from the breakdown zone, which is characterized by the intensive release of energy and a considerable increase in pressure. Ionization occurs at the shock wave front, and this makes possible the absorption of radiation as a result of the braking mechanism.

The heating of the gas which results in excitation of the atoms and ions also produces absorption (because of the photoelectric effect from highly excited states). If the gas ahead of the front is cold and nonionized, then it usually transmits radiation in the optical range. Thus, the shock wave front marks the boundary at which radiation absorption begins (i.e. it initiates absorption and energy release due to absorption). There are, of course, other factors which can produce such shock waves (electrical discharges, vaporization of the surface of a solid body under one type of radiation or another, etc.).

Absorption of radiation at small distances from the shock wave front produces a detonation wave [1].

If the radiation flux incident on the shock wave front varies, then the detonation wave propagates with a variable velocity. It is of interest to consider gas motions behind the fronts of such shock waves. With a power law of variation of the radiation flux with time, $q \sim t^{\alpha}$, the velocity of the detonation wave also varies according to a power law, and the problem is self-similar.

It is also interesting to consider gas motion in cases where the radiation is absorbed at distances comparable with characteristic dimension of the problem, and even at distances such that the radiation passes almost freely through the heated gas behind the shock wave front (through optically thin gas layers).

1. The radiation absorption coefficient $\varkappa$ due to free-free electron transitions in the ionic field depends on the temperature and density in the complete-ionization zone in the following way:

$$\varkappa \sim \varepsilon^{-2} T^{-7/2} \rho \sim \varepsilon^{-2} p^{-7/2} \rho^{9/2}$$

Here ε is the quantum energy, T is the temperature, p is the pressure, and ρ is the density.

In the multiple-ionization zone where absorption also occurs by way of the photoelectric effect from highly excited atomic and ionic states the function $\varkappa(T, \rho)$ can also be approximated by means of a power function,

$$\varkappa = k_q \rho^{-a} p^{b} = k_q v^{a} p^{b} \tag{1.1}$$

Here k_q is a numerical coefficient and v is the specific volume.

For an arbitrary optical thickness of the gas behind the shock wave front the problem is self-similar only in the event of a power dependence of the radiation flux q on the time, i.e. only if $q \sim t^{\alpha}$; moreover, the exponent α must have a certain specific value (in the case of a completely ionized gas, when $b = -5/2$, $a = -5/2$, we have $\alpha = 5/3$).

Self-similarity also requires, of course, that the piston pushing the gas move according to a certain power law $u \sim t^{g}$, where g is related to α (for $\alpha = 5/3$ we have $g = 1/3$). However, since the gas is receiving energy from an external source (radiation absorption), it is possible to have motion with a stationary piston, a piston moving backwards (according to the same power law), and even with escape into a vacuum (this is a self-maintaining, though not necessarily a detonational, state).

In the self-similar problem the optical thickness of the gas layer heated by the shock wave is always constant. However, by varying the parameters of the problem (e.g. the initial density ρ_0 of the medium in which the shock wave propagates) it is possible to trace the changes in the pattern of motion accompanying the transition from one limiting case to another, i.e. from a detonation state in which all the energy is released almost directly behind the shock wave front, to a state in which the gas absorbs only a small part of the radiant energy incident on the wave front.

Let us consider the case of a low initial density where the region behind the shock wave absorbs the incident radiation weakly, and where we can neglect the change in the luminous flux in the heated region in comparison with its value at the shock wave, i.e. where we can set

$$r^{\delta} q(m, t) = r^{\delta}_0 \; q_0 = \text{const} \tag{1.2}$$

The exponent α in this problem is arbitrary. For simplicity, we shall carry out our computations for $\alpha = 0$.

The system of equations describing the motion of the gas with allowance for radiation absorption is in this case of the form

$$\frac{\partial u}{\partial t} + r^{\nu-1}\frac{\partial p}{\partial m} = 0, \qquad \frac{\partial r}{\partial t} = u, \qquad \frac{\partial r^{\nu}}{\partial m} = \nu v$$
$$v\frac{\partial p}{\partial t} + \gamma p \frac{\partial v}{\partial t} = (\gamma - 1) q_0 k_q \left(\frac{r_0}{r}\right)^{\delta} v^{a} p^{b} \tag{1.3}$$

Here u is the velocity, m is the mass Lagrange coordinate, γ is the adiabatic exponent, $\nu = 1,2,3$ in the place, cylindrical, and spherical cases, respectively, $\delta = 0$ for an unfocused beam, $\delta = \nu - 1$ for a focused beam, and r is the Euler coordinate.

Naturally, relation (1.2) and therefore the solution of the problem in the case of focused radiation, is valid only for those instants when the radius of the shock wave considerably exceeds the minimum focusing radius of the incident radiation

Let the shock wave move initially through a cold, stationary, homogeneous gas,

$$p(m, 0) = u(m, 0) = 0, \qquad v(m, 0) = v_0 = \text{const} \tag{1.4}$$

We introduce the self-similar variables V, P, U, R, x given by the formulas

$$v = v_0 V, \qquad p = t^{-2(c/\nu+1)} v_0^{(2-\nu-2f)/\nu} (k_q q_0 r_0^{\delta})^{-2d/\nu} P \tag{1.5}$$
$$u = t^{-1-c/\nu} v_0^{(1-f)/\nu} (k_q q_0 r_0^{\delta})^{-d/\nu} U, \qquad r = t^{-c/\nu} v_0^{(1-f)/\nu} (k_q q_0 r_0^{\delta})^{-d/\nu} R$$
$$x = m t^{c} v_0^{f} (k_q q_0 r_0^{\delta})^{d}$$
$$f = \frac{a\nu + 2b - b\nu - 2 - \delta}{2(b-1) - \delta} \qquad c = \frac{(3-2b)\nu}{2(b-1)-\delta}, \qquad d = \frac{\nu}{2(b-1) - b}$$

Substituting them into (1.3), we obtain the system of self-similar equations

$$cxU' + R^{\gamma-1} P' = (1 + c / \nu)U, \qquad R^{\gamma-1} R' = V \tag{1.6}$$
$$c\,(xR' - R / \nu) = U$$
$$cx\,(VP' + \gamma PV') = 2\,(1 + c / \nu)\,PV + (\gamma - 1)R^{-\delta}V^{a}P^{b}$$

The boundary conditions of the problem are either the pressure at the piston or its velocity,

$$P\,(0) = P_0 \quad \text{or } (U\,(0) = U_0) \tag{1.7}$$

and the relations at the strong shock wave front.

$$V = \frac{\gamma - 1}{\gamma + 1}, \quad R = (\nu x_1)^{1/\nu}, \quad U = -\frac{2cx_1R^{\nu-1}}{\gamma + 1}, \quad P = \frac{2}{\gamma + 1}\left(\frac{cx_1}{R^{\nu-1}}\right)^2 \tag{1.8}$$

Here x_1 is the self-similar coordinate of the shock wave chosen in the course of solution of the problem in such a way as to ensure that $P(0) = P_0$ (or $U\,(0) = U_0$). In the case of escape into a vacuum ($P_0 = 0$) or of a symmetric problem ($U_0 = 0$) the single set of self-similar profiles of problem (1.6) - (1.8) yields the solution for all the parameters k_q, q_0, v_0 for which assumption (1.1) holds.

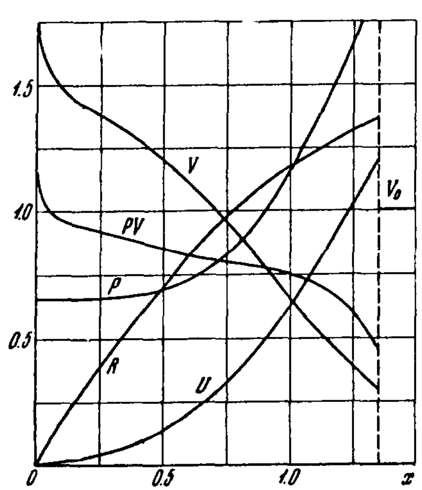

Fig. 1

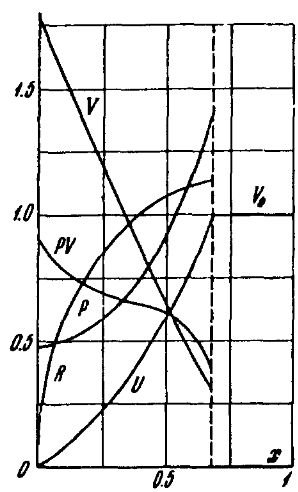

Fig. 2

Figure 1 shows the distributions of V, P, U, R, PV with respect to the self-similar variable x for the plane case with $\alpha = 0$, $a = -5/2$, $b = -3/2$, $\gamma = 5/3$ for $U_0 = 0$, for a stationary piston (the motion is maintained solely by energy release through absorption). Figs. 2 and 3 show the same functions in the case of cylindrical symmetry for focused ($\delta = -1$) and unfocused ($\delta = 0$) beams.

2. For an arbitrary optical thickness the system of equations describing the motion of a gas with allowance for radiation absorption in the plane case is of the form

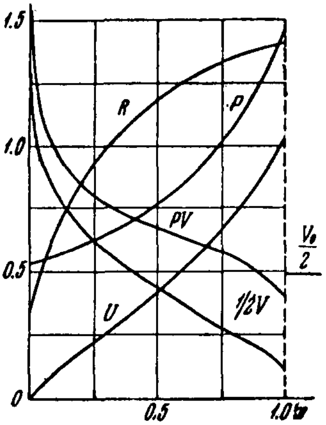

Fig. 3

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \quad \frac{\partial v}{\partial t} = \frac{\partial u}{\partial m}, \quad \frac{\partial q}{\partial m} = k_q v^a p^b q \tag{2.1}$$

$$v \frac{\partial p}{\partial t} + \gamma p \frac{\partial v}{\partial t} = (\gamma - 1) \frac{\partial q}{\partial m}$$

Let us introduce the self-similar variables V, P, U, Q, x by means of the formulas

$$v = t^{-[3+\alpha(2b+1)]/c} q_0^{-(2b+1)/c} k_q^{-3/c} V$$

$$p = t^{[1+\alpha(2a-1)]/c} q_0^{(2a-1)/c} k_q^{1/c} P \tag{2.2}$$

$$u = t^g q_0^{(a-b-1)/c} k_q^{-1/c} U, \quad q = q_0 t^\alpha Q$$

$$x = m t^s q_0^{-(a+b)/c} k_q^{-2/c}, \quad c = 3a - b - 2$$

$$g = [\alpha (a - b - 1) - 1] / c, \quad s = [b - 3a - \alpha (a + b)] / c$$

System (2.1) becomes

$$sx (VP' + \gamma PV') + \{1 - 3\gamma + \alpha[2a - 1 - \gamma(2b + 1)]\} PV / c =$$
$$= (\gamma - 1) QV^a P^b, \quad Q' = QV^a P^b$$
$$P' + sxU' + [\alpha(a-b-1) - 1] U / c = 0 \tag{2.3}$$
$$sxV' - [3 + \alpha(2b + 1)] V / c = U'$$

The condition $v (m, 0) = v_0$ yields

$$\alpha = -3 / (2b + 1) \tag{2.4}$$

The initial and boundary conditions become a system of boundary conditions for Equations (2.3),

$$V (\infty) = V_0, \quad U (\infty) = P (\infty) = 0, \quad Q (\infty) = 1$$
$$P (0) = P_0 \quad (\text{or } U (0) = U_0) \tag{2.5}$$

The relations at any strong discontinuity which may arise in the problem are of the form

$$V_2 = \frac{\gamma (P_1 + s^2 x_1^2 V_1) - [(s^2 x_1^2 V_1 - \gamma P_1)^2 - 2 (\gamma^2 - 1) s x_1 (Q_2 - Q_1)]^{1/2}}{(\gamma + 1) s^2 x_1^2}$$

$$P_2 = P_1 + s^2 x_1^2 (V_1 - V_2), \quad U_2 = U_1 + s x_1 (V_2 - V_1) \tag{2.6}$$

Where the subscripts 1 and 2 refer to quantities to the right and left of the discontinuity coordinate $x = x_1$, respectively. We note that $Q_2 \neq Q_1$ only in the case of the limiting (detonation) problem.

The presence of a free parameter (the self-similar strong discontinuity coordinate) enables us to satisfy five boundary conditions (2.5) for four equations (2.3).

According to our formulation of the initial problem, the shock wave moves through cold stationary gas, initiating the absorption of radiation. If its self similar coordinate is $x = x_1$, then

$$V = V_0, \quad P = U = 0, \quad Q = 1 \quad \text{for } x > x_1 \tag{2.7}$$

Problem (2.3) - (2.5) can be solved by choosing x_1 in such a way that $P(0) = P_0$ (or $U(0) = U_0$). However, as will be shown below, this can be done only for $P_0 > P_*(V_0)$ (or $U_0 > U_*(V_0)$). The asterisk subscript indicates certain "critical" values.

3. Numerical computations of system (2.3) under boundary conditions (2.5) in the general case of a finite nonzero V_0 show that a solution continuous over the interval $(0, x_1)$ with a single shock wave at the point $x = x_1$ can be constructed only for some (not all) P_0. The qualitative behavior of the integral curves of system (2.3) in the plane x, P is shown in Fig. 4. The solid curves represent the solutions of boundary value problem (2.3) - (2.5) for various P_0; the broken curves represent the integral curves and those portions of the latter which do not satisfy boundary conditions (2.7).

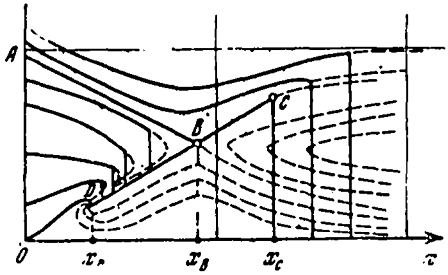

Fig. 4

The character of the singular points D and B can be determined by reducing system (2.3) to within quantities of a higher order of smallness in the neighborhood of each singular point to a single equation of the form

$$\frac{dv}{dh} = \frac{a_{11}v + a_{12}h}{a_{21}v + a_{22}h} \tag{3.1}$$

where $h = x - x_p$, x_p is the coordinate of the singular point, and the quantities a_{ik} and x_p can be expressed in terms of the values of the functions V_p, P_p, U_p, Q_p at this point by means of the formulas

$$a_{11} = \frac{1}{k}(3s^2x_p^2V_p - P_p - sx_pU_p) - (\gamma - 1)Q_pV_p^aP_p^b\left(\frac{a}{V_p} - \frac{bs^2x_p^2}{P_p}\right)$$

$$a_{12} = -\frac{U_pV_p}{k}\left(s + \frac{2}{k}\right) - (\gamma - 1)Q_pV_p^aP_p^b\left(V_p^aP_p^b + \frac{bU_p}{kP_p}\right) \tag{3.2}$$

$$a_{21} = -(\gamma + 1)s^2x_p^2, \qquad a_{22} = \frac{\gamma U_p}{k} - 2s^2x_pV_p$$

$$x_p = (\gamma P_p / V_p)^{1/2}, \qquad k = 1 + 2b$$

A characteristic feature of this problem is the existence of solutions with two discontinuities and a singular point of the saddle-point type (point B in Fig. 4) between them. The singular point D is a logarithmic node.

All of the solutions with $P_0 > P_A$ which lie above the curve ABC contain only one discontinuity whose self-similar coordinate is $x > x_c$ and depends on P_0. One such solution with $V_0 = 2$, $U_0 = 0$, $a = -{}^5/_2$, $b = -{}^3/_2$, $\gamma = {}^5/_3$ is shown in Fig. 5. The solutions with $P_0 \leqslant P_A$ have one discontinuity in common at the point $x = x_c$, pass through the singular point B, and experience a second discontinuity (which may be weak for sufficiently small P_0) on the segment $[x_D, x_B]$ whose coordinate now depends on P_0.

We therefore see that "the acoustic point B does not transmit perturbations travelling from the left-hand edge" (from the point $x = 0$), and that the first shock wave which initiates absorption propagates for all values of P_0 from the interval $(0, P_A)$ with a single velocity which depends only on the initial gas density ρ_0 and on the intensity q_0 of the incident light beam. Although the situation is in this case analogous to the propagation of a detonation wave, the analogy is not complete, since the energy in the case under consideration is released at some (possibly considerable) depth, rather than in an infinitely thin layer behind the shock wave front. Only as $V_0 \to \infty$ does the quantity $(x_B - x_c)$ tend to zero, the points B and C drawing closer together. In the limiting case the configuration consisting of the singular point B and the shock wave at the point

$x = x_C$ becomes a single detonational discontinuity (2.6) with $Q_2 = 0$ and the coordinate

$$x_d = -\frac{[2(\gamma^2 - 1)V_0^{-\gamma}]^{1/s}}{s} \tag{3.3}$$

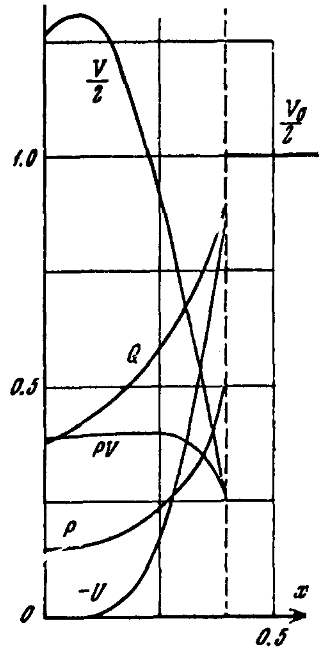

Fig. 5

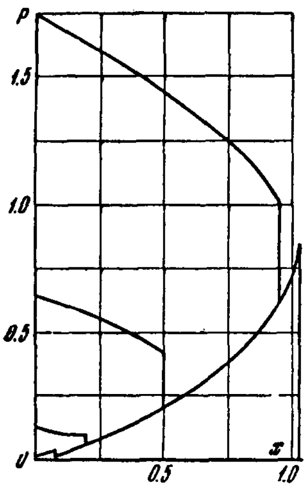

Fig. 6

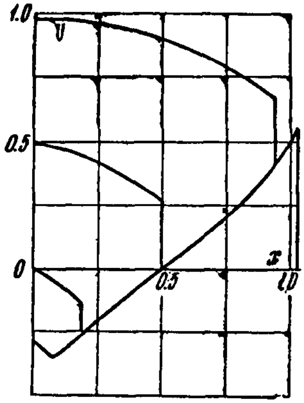

Fig. 7

The second discontinuity with the coordinate $x < x_d$, which depends on the value of P_0, is preserved. The self-similar profiles in detonational states of absorption wave propagation for several P_0 are shown in Figs. 6 and 7 ($a = -5/2$, $b = -3/2$, $\gamma = 5/3$, $\alpha = 3/2$, $V_0 = 1$).

Let us cite some values of the pressure P_0 at the point $x = 0$, and also the values of the self-similar coordinate x_s in the case of a stationary piston ($U_0 = 0$) for $a = -5/2$, $b = -3/2$, $\gamma = 5/3$ for several values of the initial specific volume V_0:

$V_0 =$	50	10	5	2	1.55
$P_0 =$	0.00219	0.0206	0.0518	0,156	0.195
$x_s =$	0.00880	0.0600	0.0135	0,392	0.518

The quantities $p(0, t)$ and $m_s(t)$ in the same case can be determined from the relations

$$p(0, t) = tq_0^{3/4} kq^{-1/8} P_0(V_0)$$
$$m_s(t) = x_s(V_0) t^{3/2} q_0^{1/2} k_q^{-1/4} \tag{3.4}$$

Recalling the function $P_0(V_0)$ in the limiting cases, we obtain

for $V_0 \ll 1$

$$p(0, t) = 0.112 t q_0^{3/4} v_0^{-1/2}$$
$$m_s(t) = 1.02\, t^{3/2} q_0^{1/2} v_0^{-3/2} \tag{3.5}$$

for $V_0 \gg 1$

$$p(0, t) = 0.513\, t q_0^{3/4} k^{-1/8} v_0^{-7/8}$$
$$m_s(t) = 0.811 t^{3/2} q_0^{1/2} k_q^{-1/4} v_0^{-9/8} - \tag{3.6}$$

4. We have not yet dealt with the question of what causes the propagation of the shock wave; we have simply assumed the existence of a certain piston. Let us now consider this matter and the formulation of problems in the absence of a piston.

Let the surface of a solid be acted on by a radiation flux which vaporizes this surface. The dispersing vapor constitutes the piston which pushes against the gaseous medium surrounding the body. If the amplitude of the resulting shock wave is sufficiently large, then marked ionization takes place at its front. Radiation absorption begins not at the vapor boundary, but rather at the shock wave front. Some of the radiation passes through the gas heated in the shock wave and continues to vaporize the material and to heat the vapor. Let the vapor be heated to such a degree that we can neglect the heat of vaporization Q_v in comparison with the enthalpy for the maximum vapor temperature T_m and also the phase transition temperature T_v as compared with T_m.

If the absorption coefficients of the vapor and medium depend exponentially on the temperature and density and if the exponents for the vapor and medium are equal (e.g. if both the vapor and the medium behind the shock wave are completely ionized), then the problem is self-similar. The problem has already been considered [2,3] in the absence of a medium with a shock wave propagating through it (i.e. in the case of dispersion into a vacuum).

The same approach can be used to consider the action of radiation on an initially cold stationary gas in which strong shock waves arise at the boundary of two media of differing densities ρ_1 and ρ_2 (or simply at some point of a homogeneous medium in the special case where these two densities are equal). If their amplitude is sufficiently large, then the gas behind them is ionized and begins to absorb radiation incident on the fronts of these shock waves (if the sources are situated on opposite sides) or on the front of one of the shock waves (if there is only one source).

The initial push which initiates these waves can be produced by detonating a piece of foil or vaporizing a piece of foil by irradiation. With an initial "push" of sufficient amplitude to give rise to shock waves and radiation absorption behind them, the parameters of motion do not depend on the properties of this "detonator" for times much larger than the initiation time, since the total energy contained in the region between shock waves and released there as a result of absorption of radiation from the external source is much larger than the initial detonator energy. This problem is self-similar. Self-similarity conditions (2.2) are preserved in this case. Equations (2.3) are of the same form. The boundary conditions are

$$V(\infty) = V_1, \qquad V(-\infty) = V_2, \qquad Q(\infty) = 1$$
$$U(\infty) = U(-\infty) = P(\infty) = P(-\infty) = 0 \tag{4.1}$$

(we assume for simplicity that the radiation is coming from one side only).

The problem of vaporization of a solid with allowance for the resistance to vapor dispersion and radiation absorption by the medium can be considered as the special case of the problem for $V_2 = 0$.

It is clear that this problem involves two boundary conditions more than the problem of Sect. 2. One of the free parameters which can be used to satisfy one of the additional boundary conditions is the self-similar coordinate of the shock wave travelling through the dense medium (in the same direction as the light beam) and initiating radiation absorption in the latter. As the second free parameter we can take the jump in density at the point $x = 0$ (as will be shown below, however, a contact discontinuity is impossible).

If the point $x = 0$ is singlular, i.e. if

$$(\gamma - 1)\, Q(0) V^{a-1}(0) P^{b-1}(0) = -2/k \tag{4.2}$$

and if $P(0) > 0$, then the solution in the neighborhood of this point is of the form

$$V = V_0 + A|x|^{(a-1)/b\gamma} + \frac{2P_0^{1+b}V_0^{1+a} + U_0V_0(2bk^{-1} - 2k^{-1} - s)}{2P_0(a - 1 - b\gamma)}\, x$$

$$U = U_0 - sx(V - V_0), \qquad P = P_0 + U_0 x/k, \qquad Q = Q_0(1 - V_0^a P_0^b x) \tag{4.3}$$

to within quantities of a higher order of smallness.
Here A is an arbitrary constant and the zero subscript refers to quantities at the point $x = 0$.
If $P(0) = 0$, then the singular point $x = 0$ is also a node, and its solution in its neighborhood to within quantities of a higher order of smallness is of the form

$$V = \left[\frac{(1-\gamma) Q_0 (U_0 |x| / k)^{b-1}}{s[1+\gamma(b-1)/(a-1)] + 2/k} + A|x|^{\theta}\right]^{1/(1-a)}$$

$$U = U_0 + sxV(1-b)/(b-a), \qquad P = U_0 x / k$$

$$Q = Q_0\left[1 - \left(\frac{U_0}{k}\right)^b |x|^{b+1} V^a \frac{1-a}{b-2a+1}\right] \tag{4.4}$$

$$\theta = (1-a)(s - 2k^{-1}) / \gamma s$$

Let us assume that there exists a solution such that

$$P_0 > 0, \qquad V_0 < \infty, \qquad (\gamma - 1) Q_0 V_0^{\,a-1} P_0^{\,b-1} \neq -2/k \tag{4.5}$$

In this case we obtain the following approximate equation for determining $V(x)$ in the neighborhood of the point $x = 0$:

$$\frac{dV}{dx} = \frac{(1-\gamma) Q_0 V_0^a P_0^b - 2P_0 V_0 / k}{\gamma P_0 s x} = \frac{A_1}{x} \tag{4.6}$$

which yields

$$V = A_1 \ln(cx) \tag{4.7}$$

This contradicts our initial assumption (4.5). This means that if relation (4.2) is not fulfilled at the point $x = 0$, then

$$V(0) = \infty, \ P(0) = 0 \tag{4.8}$$

Let us assume that there is a contact discontinuity at the point $x = 0$ i.e. that

$$V(+0) \neq V(-0), \qquad P(+0) = P(-0), \qquad U(+0) = U(-0),$$
$$Q(+0) = Q(-0) \tag{4.9}$$

It is clear that at least one of the points $x = +0$ or $x = -0$ in this case is non-singular, i.e. such that relation (4.2) is not fulfilled there. Let this be the point $x = +0$. Then according to the above result (4.8), we have

$$V(+0) = \infty, \qquad P(+0) = 0 \tag{4.10}$$

According to (4.9) it is also the case that $P(-0) = 0$, and from (4.8) we find that $V(-\infty) = \infty$.
We therefore conclude that there is no contact discontinuity, i.e. that

$$\rho(+0) = \rho(-0) = 0 \tag{4.11}$$

Q.E.D.
If $V(0) < \infty$, $P(0) > 0$, then a shock discontinuity is also impossible at the point $x = 0$, since

$$V_2 = V_1(\gamma-1)/(\gamma+1) + 2\gamma P_1 / [(\gamma+1) s^2 x^2], \qquad Q_2 = Q_1$$
$$P_2 = P_1 + s^2 x^2 (V_1 - V_2), \qquad U_2 = U_1 + sx(V_1 - V_2) \tag{4.12}$$
$$V_2 = \infty \quad \text{for} \quad x = 0$$

The case $P(0) = 0$ again yields

$$V(+0) = V(-0) = \infty \quad (\rho(+0) = \rho(-0) = 0)$$

i.e. there is no contact discontinuity; if $P_1(0) = 0$, then $V_1(0) = \infty$, and

$$V_2 = \frac{\gamma - 1}{\gamma + 1} V_1 + \frac{2\gamma P_1}{(\gamma + 1) s^2 x^2} = \infty$$

Thus, the second free parameter is the quantity A in expression (4.3) or (4.4), and a weak discontinuity exists at the point $x = 0$.

A sample solution of the problem of an absorption wave at the boundary between two media of differing densities appears in Fig. 8 for the case $a = -{}^5/_3$, $b = -{}^3/_2$, $\gamma = {}^5/_3$, $V_2 = 0$, $V_1 = 45$. The dots represent parameters at the shock wave propagating in the direction opposite to the oncoming radiation flux.

Figure 9 shows the pressure P_v at the forward edge of the heating wave and at the shock wave, which has travelled into the interior of the material, as a function of the initial density V_1 of the medium. The same figure shows the behavior of $P(0) = P_0$, i.e. the pressure at the stationary boundary for the problem of motion of the absorption-initiating shock wave away from this surface ($U(0) = 0$).

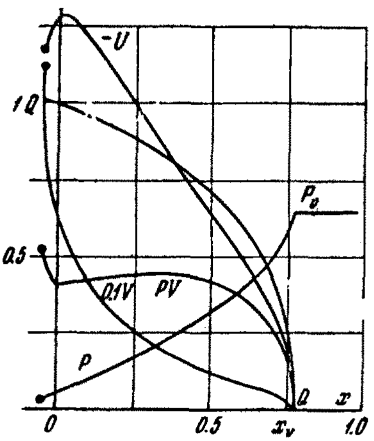

Fig. 8

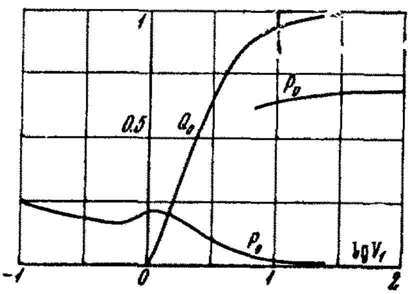

Fig. 9

The quantities P_v and $P(0)$ practically coincide for high densities of the medium ($V_1 \leqslant 1$). This is because practically no radiation passes through it (this is obvious from Fig. 9, which shows the dependence of the radiation flux $Q(0) = Q_0$ at the stationary boundary of the medium). For very high-density media ($V_1 \ll 1$) both quantities P_v and $P(0)$ merge with the function given by the solution of the problem of propagation of a detonation wave with a variable detonation rate (when all of the energy is absorbed in a narrow layer near the shock wave front),

$$P_v = P(0) = A(\gamma) V_1^{-1/3} \qquad (A = 0.112 \text{ при } \gamma = {}^5/_3) \tag{4.13}$$

According to Fig. 9 and relation (4.13) the pressure increases with increasing density in a detonation state and in a state such that the mass of the energy release zone is comparable to the entire mass encompassed by the shock wave front, but with the medium still practically opaque. On the other hand, in the case of low densities ($V_1 \gg 1$) the pressure decreases with increasing density.

This is attributable to the fact that the pressure drop is due solely to the dispersion of the radiation-heated "vapor", while the medium is almost 100% transparent and resists dispersion only slightly. The slight absorption in the medium reduces the energy supplied to the vapor (this screening effect reduces the vaporization intensity and the pressure at the "vaporized surface"). The minimum pressure is attained with dimensionless densities of the medium on the order of unity, i.e. for densities of the medium close to the average vapor density in the case of dispersion into a vacuum [8].

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STATISTICAL MECHANICS OF GAS SUSPENSIONS. A QUASI-ISOTROPIC MODEL

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A statistical theory of streams of systems of the gas + suspended particles type is proposed. The theory is based on the assumption that the particle concentration fluctuations are isotropic. The structure of the equilibrium states is considered in a gradientless approximation; the mean-square values of the pulsations of the dynamic quantities and the transfer coefficients are estimated; the size of the local inhomogeneities is determined. Equations for the energy of the pulsations of the dispersed phase in various directions are obtained; an energy equation complementing the system of dynamic equations given in [1] is derived.

The statistical characteristics of random pulsating motions of the phases in a gas suspension stream can be found by solving the system of integrodifferential spectral equations obtained in [1]. This system is quite complicated. It is therefore expedient to make use of some simplifying hypotheses; this makes it possible to reduce the stochastic equations of [1] to the equations of [2].

1. The pulsation equations. Let us consider the motion of a monodisperse gas suspension under the assumption that the time and space scales of variation of the mean parameters describing the flow (of the "dynamic quantities") are large as compared with the scales of the local pulsations. This enables us, among other things, to carry out our computations in a coordinate system in which the velocity of the dispersed phase in the volume element under consideration is equal to zero.

Let us make use of the most "fine-grained" description of the pulsations of dynamic quantities permitted by the notion of phases as interacting interpenetrating continuous media, i.e. let us choose as our characteristic physical volume (the "averaging scale" in the terminology of [1] the specific volume $\sigma = l^3$ of a single suspended particle [1,2]. In accordance with the above assumption we neglect the derivatives of the dynamic quantities with respect to time and the coordinates as compared with the corresponding derivatives of the fluctuations of these quantities. This approximation is analogous in meaning to the familiar hydrodynamic approximation of kinetic theory [2]. We then have the following equations for the pulsations of the mean parameters (the notation is that of [1]

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\frac{\partial}{\partial \mathbf{r}}\right)\rho_l' - (1-\rho)\frac{\partial \mathbf{v}_l'}{\partial \mathbf{r}} = Q_l^{(g)}, \qquad \frac{\partial \rho_l'}{\partial t} + \rho\frac{\partial \mathbf{w}_l}{\partial \mathbf{r}} = Q_l^{(p)}$$

$$-(1-\rho)\frac{\partial \pi_l'}{\partial \mathbf{r}} - \left(-\frac{d\pi}{d\mathbf{r}} + \beta\frac{d(\rho K)}{d\rho}\,\mathbf{u}\right)\rho_l' - \beta\rho K(\mathbf{v}_l' - \mathbf{w}_l') = \mathbf{F}_l^{(i)}$$